ASSIGNMENT 3 - SOLUTION

Problem 1

A company that specializes in fighting wild fires in California uses two types of aircraft.

- The Bombardier CL 415 carries a maximum of 1,600 gallons of water in a single flight.
- The Air Tractor AT-802F carries 1,000 gallons per flight.
- The Air Tractor AT-802F is a single crew member plane.
- The operation of the Bombardier CL-415 requires two crew members.
- The company has 55 CL-415 and 50 AT-802F in the inventory to fight fires in the Spring 2022 season.
- The company has 135 crew members trained to operate either the CL415 or the AT 802F at its headquarters in Victorville (California).
- No more crews can be used in the fire-fighting operation.

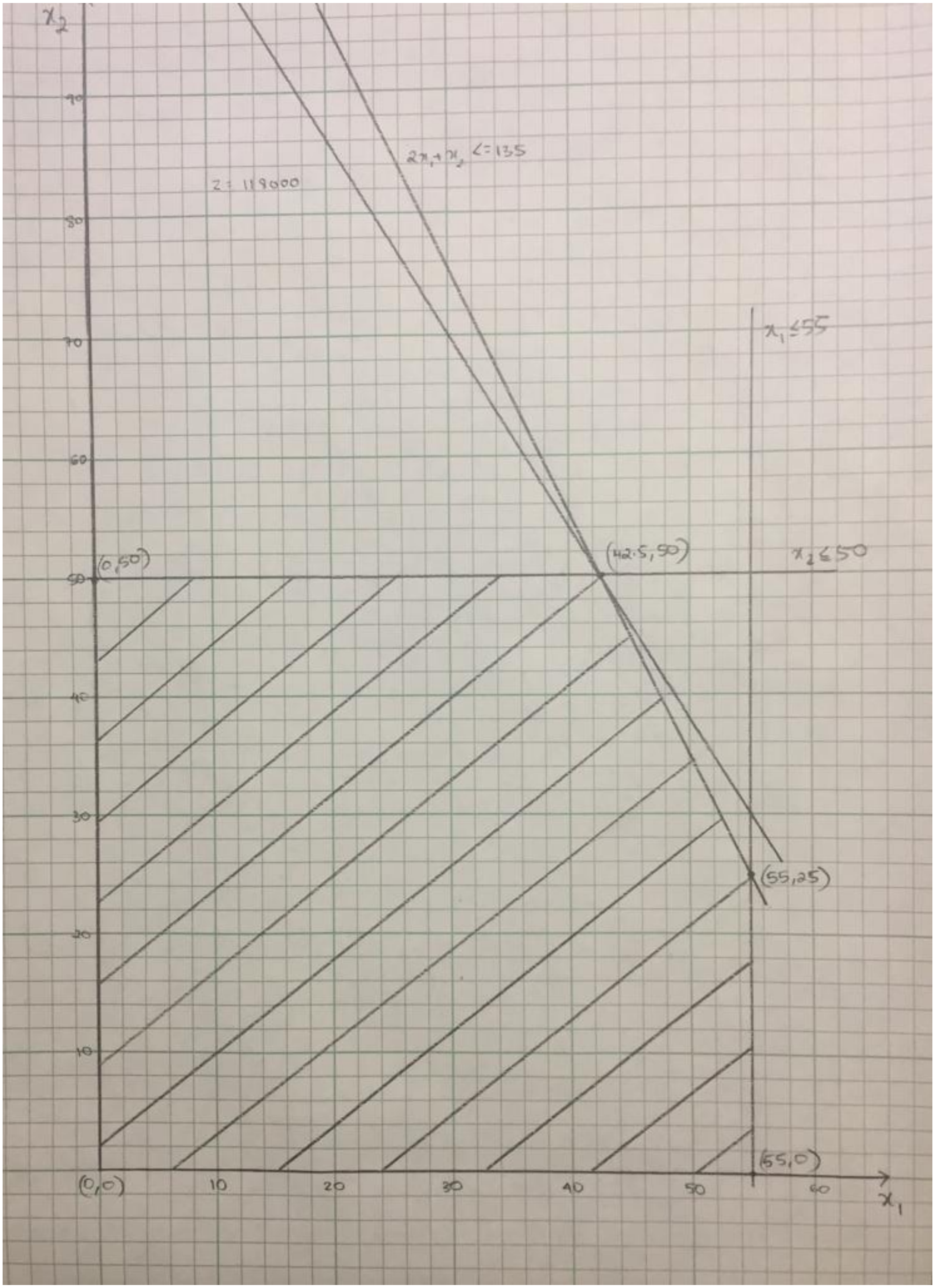
During a fire, the objective is to maximize the water load carried.
a) Formulate the problem as a linear programming problem. Clearly indicate the objective function and the functional constraints.
b) Solve the problem graphically. Clearly indicate corner points and plot two lines of constant Z value at two corner points.
c) Solve the problem using the Simplex Method. Make sure you show me every table in the procedure. For each table indicate the Basic Variables, Non-basic variables and value of the objective function (Z).
d) Solve the problem using Excel Solver. Verify the answer obtained in Part (c).
Ans.

a) Decision Variables
   x1 – number of the Bombardier CL 415
   x2 – number of the Air Tractor AT-802F

   Objective Function
   Z = 1600x1 + 1000x2

   Constraints
   2x1 + x2 <= 135
   x1 <= 55
   x2 <= 50
   x1 >= 0
   x2 >= 0

b) Graph
   Shaded region is the feasible region
   Corner Points: (0,0) (0,50) (42.5,50) (55,25) (55,0)


x_2
90
80
70
60
50
40
30
20
10
$2x_1+x_2 <= 135$
Z = 119000
$x_1 \leq 55$
(0,50)
(42.5, 50)
$x_2 \leq 50$
(55,25)
(55,0)
(0,0)
10
20
30
40
50
60
x_1

c) Simplex Method

Standard Form

Z - 1600x1 - 1000x2 = 0

2x1 + x2 + x3 = 135

x1 + x4 = 55

x2 + x5 = 50

x1,x2,x3,x4,x5 >= 0

First Tableau

BV = x3,x4,x5 ; NBV = x1,x2

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	-1600	-1000	0	0	0	0		
x3	0	2	1	1	0	0	135	67.5	
x4	0	1	0	0	1	0	55	55	x(1600)+Z-row
x5	0	0	1	0	0	1	50	inf	

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	-1000	0	1600	0	88000

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	-1600	-1000	0	0	0	0		
x3	0	2	1	1	0	0	135		
x4	0	1	0	0	1	0	55		x(-2)+x3-row
x5	0	0	1	0	0	1	50		

x3	0	0	1	1	-2	0	25

New Tableau

Leaving BV = x4 ; New BV = x1

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	0	-1000	0	1600	0	88000		
x3	0	0	1	1	-2	0	25	25	x(1000)+Z-row
x1	0	1	0	0	1	0	55	Inf	
x5	0	0	1	0	0	1	50	50	

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	0	1000	-400	0	113000

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	0	-1000	0	1600	0	88000		
x3	0	0	1	1	-2	0	25		x(-1)+x5-row
x1	0	1	0	0	1	0	55		
x5	0	0	1	0	0	1	50		

x5	0	0	0	-1	2	1	25

New Tableau
Leaving BV = x3 ; New BV = x2

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	0	0	1000	-400	0	113000		
x2	0	0	1	1	-2	0	25	-12.5	
x1	0	1	0	0	1	0	55	55	
x5	0	0	0	-1	2	1	25	12.5	/2

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	0	0	1000	-400	0	113000		
x2	0	0	1	1	-2	0	25		
x1	0	1	0	0	1	0	55		
x5	0	0	0	-1/2	1	1/2	25/2		x(400)+Z-row

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	0	800	0	200	118000

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	0	0	1000	-400	0	113000		
x2	0	0	1	1	-2	0	25		
x1	0	1	0	0	1	0	55		
x5	0	0	0	-1/2	1	1/2	25/2		x(2)+x2-row

x2	0	0	1	0	0	1	50

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	0	0	1000	-400	0	113000		
x2	0	0	1	1	-2	0	25		
x1	0	1	0	0	1	0	55		
x5	0	0	0	-1/2	1	1/2	25/2		x(-1)+x1-row

x1	0	1	0	1/2	0	-1/2	42.5

Final Tableau
Leaving BV = x5 ; New BV = x4

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	0	800	0	200	**118000**
x2	0	0	1	0	0	1	**50**
x1	0	1	0	1/2	0	-1/2	**42.5**
x4	0	0	0	-1/2	1	1/2	25/2

Optimal Solution: (x1,x2,x3,x4,x5) = (**42.5**,**50**,0,0,10)

d) Excel Solver

Objective Function			
Z	118000		
Decision Variables			
x1	42.5	number of the Bombardier CL 415	
x2	50	number of the Air Tractor AT-802F	
Constraints			
2x1+x2	135	<=	135
x1	42.5	<=	55
x2	50	<=	50
x1	42.5	>=	0
x2	50	>=	0

Problem 2

The company in Problem 1 has a maintenance facility at Victorville. Due to heavy maintenance required on the two types of aircraft used in fire-fighting, the company estimates that a total of 85 aircraft can be made available to fight fires during the season with reliability.
a) Adding the new constraint, re-formulate the problem as a linear programming problem. Clearly indicate the objective function and the functional constraints.
b) Solve the problem using the Simplex Method. Make sure you show me every table in the procedure. For each table indicate the Basic Variables, Non-basic variables and value of the objective function (Z).
c) Solve the problem using Excel Solver.

Ans.

a) Decision Variables
x1 – number of the Bombardier CL 415
x2 – number of the Air Tractor AT-802F

Objective Function
Z = 1600x1 + 1000x2

Constraints

$x1 + x2 <= 85$ New constraint equation to account for maintenance.

$2x1 + x2 <= 135$

$x1 <= 55$

$x2 <= 50$

$x1 >= 0$

$x2 >= 0$

Note: You can force the new maintenance constraint equation to be exactly equal to 85. However, that requires using the Big-M method. You will arrive to the same answer.

b) Simplex Method

Standard Form

$Z - 1600x1 - 1000x2 = 0$

$2x1 + x2 + x3 = 135$

$x1 + x4 = 55$

$x2 + x5 = 50$

$x1 + x2 + x6 = 85$

$x1,x2,x3,x4,x5,x6 >= 0$

Final Tableau
Leaving BV = x6 ; New BV = x2

BV	Z	x1	x2	x3	x4	x5	x6	RHS
Z	1	0	0	0	600	0	1000+M	**118000**
x3	0	0	0	1	1	0	-2	20
x1	0	1	0	0	1	0	0	55
x5	0	0	0	0	1	1	1	20
x2	0	0	1	0	-1	0	1	30

Optimal Solution: (x1,x2,x3,x4,x5,x6) = (**55,30**,20,0,20,0)

c) Excel Solver

Objective Function			
Z	118000		
Decision Variables			
x1	55	number of the Bombardier CL 415	
x2	30	number of the Air Tractor AT-802F	
Constraints			
x1+x2	85	==	85
x1+2x2	115	<=	135
x1	55	<=	55
x2	30	<=	50
x1	55	>=	0
x2	30	>=	0

Problem 3

Your company develops the following Linear Programming problem to minimize the cost of producing two types of concrete pipes commonly used in buildings. The objective function is the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize Z = 65x1 + 60x2

Subject to - 1.1 x1 + x2<= 310

x1 + 6x2<= 1300
3x1 + x2 <= 600
x1, x2 >= 0 (non-negativity conditions)

a) Solve the problem using Excel Solver. State the exact solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found.
b) Solve the first two tables using the Simplex method. For each iteration, indicate the Basic Variables (in the table) and the current solution for Z.
Ans.
a) Excel Solver

Objective Function			
Z	20441.18		
Decision Variables			
x1	135.2941		
x2	194.1176		
Constraints			
(-1.1)x1+x2	45.29412	<=	310
x1+6x2	1300	<=	1300
3x1+x2	600	<=	600
x1	135.2941	>=	0
x2	194.1176	>=	0

b) Simplex Method
Standard Form
Z - 65x1 - 60x2 = 0
-1.1x1 + x2 + x3 = 310
x1 + 6x2 + x4 = 1300
3x1 + x2 + x5 = 600
x1,x2,x3,x4,x5 >= 0

First Tableau
BV = x3,x4,x5 ; NBV = x1,x2

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	-65	-60	0	0	0	0		
x3	0	-1.1	1	1	0	0	310	-281.818	
x4	0	1	6	0	1	0	1300	1300	
x5	0	3	1	0	0	1	600	200	/3

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	-65	-60	0	0	0	0		
x3	0	-1.1	1	1	0	0	310		
x4	0	1	6	0	1	0	1300		
x5	0	1	1/3	0	0	1/3	200		x(65)+Z-row

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	-115/3	0	0	65/3	13000

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	-65	-60	0	0	0	0		
x3	0	-1.1	1	1	0	0	310		
x4	0	1	6	0	1	0	1300		
x5	0	1	1/3	0	0	1/3	200		x(1.1)+x3-row

x3	0	0	4.1/3	1	0	1.1/3	530

BV	Z	x1	x2	x3	x4	x5	RHS	ratio	
Z	1	-65	-60	0	0	0	0		
x3	0	-1.1	1	1	0	0	310		
x4	0	1	6	0	1	0	1300		
x5	0	1	1/3	0	0	1/3	200		x(-1)+x4-row

x4	0	0	17/3	0	1	-1/3	1100

New Tableau
Leaving BV = x5 ; New BV = x1

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio	
Z	1	0	-115/3	0	0	65/3	13000		
x3	0	0	4.1/3	1	0	1.1/3	530	387.805	
x4	0	0	17/3	0	1	-1/3	1100	194.118	x(3/17)
x1	0	1	1/3	0	0	1/3	200	600	

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio	
Z	1	0	-115/3	0	0	65/3	13000		
x3	0	0	4.1/3	1	0	1.1/3	530		
x4	0	0	1	0	3/17	-1/17	3300/17		x(115/3)+Z-row
x1	0	1	1/3	0	0	1/3	200		

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	0	0	115/17	330/17	347500/17

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio	
Z	1	0	-130/3	0	0	65/3	13000		
x3	0	0	4.1/3	1	0	1.1/3	530		
x4	0	0	1	0	3/17	-1/17	3300/17		x(-4.1/3)+x3-row
x1	0	1	1/3	0	0	1/3	200		

x3	0	0	0	1	4.1/17	38/85	4500/17

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio	
Z	1	0	-130/3	0	0	65/3	13000		
x3	0	0	4.1/3	1	0	1.1/3	530		
x4	0	0	1	0	3/17	-1/17	3300/17		x(-1/3)+x1-row
x1	0	1	1/3	0	0	1/3	200		

x1	0	1	0	0	-1/17	6/17	2300/17

Final Tableau
Leaving BV = x4 ; New BV = x2

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	0	0	115/17	330/17	347500/17 = **20441.18**
x3	0	0	0	1	4.1/17	38/85	4500/17
x2	0	0	1	0	3/17	-1/17	3300/17
x1	0	1	0	0	-1/17	6/17	2300/17

Optimal Solution: (x1,x2,x3,x4,x5) = (2300/17, 3300/17, 4500/17,0,0)
(**x1,x2**,x3,x4,x5) = (**135.2941, 194.1176**, 264.7059,0,0)

Problem 4

A construction site requires a minimum of 45,000 cu. meters of sand and gravel mixture.
The mixture must contain no less than 22,000 cu. meters of sand
The mixture must contain no more than 26,000 cu. meters of gravel.
Materials may be obtained from three sites:
45% sand and 55% gravel from site 1 at a delivery cost of $146 per cu. Meter
51% sand and 49% gravel from site 2 at a delivery cost of $148 per cu. Meter
52% sand and 48% gravel from site 3 at a delivery cost of $147.00 per cu. Meter
Due to limited number of excavators at each site,
Site 1 can produce up to 22,000 cubic meters of material
Site 2 can produce up to 25,000 cu. Meters
Site 3 can produce up to 27,000 cu. meters of material
a) Formulate the problem as a linear programming problem to minimize the cost to the company
b) Solve the problem using Excel Solver. Find the amounts of material to be extracted from each site
c) State the cost to produce the 45,000 cubic meters of material

Ans.

a)

	Site 1	Site 2	Site 3	Construction Site
Cost per cu.m ($)	146	148	147	
Sand (cu.m)	45%	51%	52%	>= 22000
Gravel (cu.m)	55%	49%	48%	<= 26000
Total Mixture (cu.m)	<= 22000	<= 25000	<= 27000	>= 45000

Objective Function

Z = 146x1 + 148x2 + 147x3

Decision Variables

x1 – amount of material from site 1

x2 – amount of material from site 2

x3 – amount of material from site 3

Constraints

0.45x1 + 0.51x2 + 0.52x3 >= 22000

0.55x1 + 0.49x2 + 0.48x3 <= 26000

x1 + x2 + x3 >= 45000

x1 <= 22000

x2 <= 25000

x3 <= 27000

x1,x2,x3 >= 0

b) Excel Solver

Amount of material to be extracted from

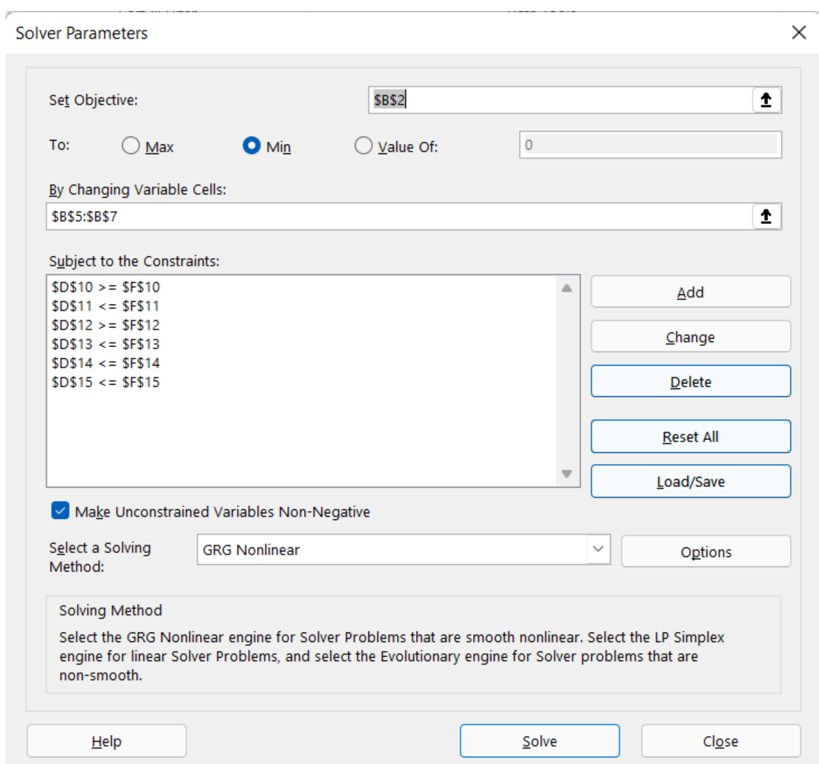
Site 1 = 20000 cu.m

Site 2 = 0 cu.m

Site 3 = 25000 cu.m

Objective Function					
Z	6595000				
Decision Variables					
x1	20000				
x2	4.55E-13				
x3	25000				
Constriants					
0.45x1 + 0.51x2 + 0.52x3			22000	>=	22000
0.55x1 + 0.49x2 + 0.48x3			23000	<=	26000
x1 + x2 + x3			45000	>=	45000
x1			20000	<=	22000
x2			4.55E-13	<=	25000
x3			25000	<=	27000
x1			20000	>=	0
x2			4.55E-13	>=	0
x3			25000	>=	0

Note: Optimal solution for x2 is zero. The value of 4.55 e-13 is a rounding error in the solution using Excel Solver.



c) Since the objective function (cost) was minimized, the excel solver will solve for the minimum amount of material required in the construction site which is 45,000 cu.m. So, from the above the cost to produce the 45,000 cubic meters of material is $6595000.